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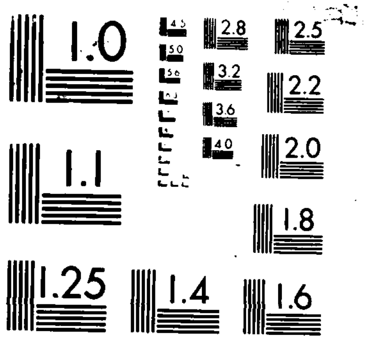

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AD-A177 715

CORRECTING ABERRATED WAVEFRONTS FROM SYNTHETIC

APERTURES HOLOGRAPHICALLY

THESIS

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Captain, USAF

DEPARTMENT OF THE AIR FORCE
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(1)

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THESIS

Presented to the Faculty of the School of Engineering

of the Air Force Institute of Technology

Air University

In Partial Fulfillment of the

Requirements for the Degree of

Master of Science in Engineering Physics

David A. Jacobs

Captain, USAF

December 1986

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AFIT/GEP/ENP/86D-4

CORRECTING ABERRATED WAVEFRONTS FROM SYNTHETIC

APERTURES HOLOGRAPHICALLY

by

David A. Jacobs
Captain, USAF

Abstract

This report describes research to investigate how well holographic optical elements reduce aberrations in synthetic aperture optical systems. Using holograms to correct aberrated wavefronts from synthetic apertures provides a viable alternative to using active control systems to perform the same function. The holographic technique has applications to the Strategic Defense Initiative, surveillance and reconnaissance systems, and high technology manufacturing processes. A model for a three-element synthetic aperture system was developed to determine how the wavefronts from the elements combine in the image plane. The model includes developing the technique for introducing a holographic optical element to correct aberrations from individual elements in the aperture and from misalignments of elements in the system. Based on this model, a proof-of-concept system was developed. Finally, the results of experiments to evaluate the point spread function of the empirical system are described. The report concludes that holographic optical elements can provide an alternative to control systems when combined with synthetic apertures to replace monolithic optics in applications requiring diffraction limited imaging systems.

CORRECTING ABERRATED WAVEFRONTS FROM SYNTHETIC

APERTURES HOLOGRAPHICALLY

I. Introduction

Problem

The purpose of this research was to investigate how well holographic optical elements can reduce aberrations in the image plane of synthetic aperture optical systems. Synthetic apertures are a developing technology with applications to the Strategic Defense Initiative, surveillance and reconnaissance systems, and high technology manufacturing procedures. A synthetic aperture is a device which uses one or more small apertures to obtain the resolution normally associated with a single large aperture (Goodman (1970), p 3). An aperture is defined here to mean a transmissive part in an opaque screen, with the implications that the aperture is filled with a combination of lenses, mirrors, and prisms. Aberrations can arise from elements in the aperture, or from misalignments of individual elements in the system.

This investigation required three steps. The first step was to analytically describe the system under consideration. Having described the system, the second step was to propose suitable holographic optical elements to correct potential aberrations in the described system. The final step was to test the proposed holographic optical elements for the described system.

Background

The Air Force needs imaging systems with high resolving capabilities in three critical mission areas. Surveillance, Acquisition, Tracking, and Kill Assessment (SATKA) missions require optical systems with high resolving capabilites to support the Strategic Defense Initiative. Reconnaissance missions require imaging systems with high resolving capabilities to support strategic and tactical intelligence at national and operational levels. And, manufacturing Very Large Scale Integrated Circuits (VLSIC) requires imaging systems with high resolving capabilites to produce integrated circuits.

The Strategic Defense Initiative (SDI) requires SATKA assets to detect launch of missiles, acquire and track their warheads, and assess the effectiveness of defensive actions. Optical systems are being proposed for this task. As with other missions, optical systems designed to respond to SDI needs using traditional technologies are too large and too massive to be practical. Synthetic apertures may offer an alternative to monolithic optics if fundamental problems of aligning the optical system can be solved.

Strategic and tactical intelligence requirements drive reconnaissance missions. For example, President Reagan used imagery intelligence to explain why the United States participated in invading Grenada (Time, 7 Nov 83). As another example, President Reagan used imagery following the attacks against Libya to demonstrate how selective the United States was in destroying military targets (Time, 28 Apr 86). As with the SDI mission, synthetic apertures may allow greater resolution than current systems, while also reducing overall costs.

Producing densely populated VLSIC components requires high resolution optical systems. The actual circuit is produced from a photoreduced drawing of the circuit. The density with which components can be packed into the circuit is partly dependent on the resolution of the photoreducing system. Active research may allow a new generation of integrated circuits to pack components more densely, requiring photoreducing systems with new levels of resolving capabilites. Moreover, higher density VLSIC components may have applications in upgrading SATKA and Reconnaissance assets.

Synthetic Aperture Optical Systems

Designing diffraction limited optics requires designing an optical system and then correcting for aberrations. Technology exists to design and construct diffraction limited optics, but even with small optics, costs are high; large optics are impracticable in sufficient quantities for Air Force missions. Reducing diffraction effects requires larger optics, but larger optics are massive, dimensionally large, and expensive. One way to create a diffraction limited optic without these disadvantages is to construct a large optic from many smaller optics. Such an optic is called a synthetic aperture or multi-aperture optic. As an example of how synthetic aperture technology is already used, the Multi-Mirror Telescope at the University of Arizona has a six-element synthetic aperture main mirror (Meinel). The resulting optic will be relatively light and inexpensive, but focusing the resulting image requires phasing the wavefronts from individual optical elements. Additionally, compensating for piston and tilt misalignments in the assembled optic requires rephasing the wavefronts from the ele-

ments. One method of correcting the phase of an aberrated wavefront is to operate on the wavefront with a hologram.

Holograms are optical devices which store both amplitude and phase information. Holograms differ from ordinary photographs, which store only information describing the intensity of the sampled wavefront, because holograms have the property that an incident beam can reproduce the recorded wavefront. One way to store amplitude and phase information is to use the principles of interferometry to map phase information into intensity information, and then to store the intensity information in a photographic plate. The photographic plate is then called a hologram. By using the hologram as a diffraction grating, the hologram acts as a wavefront correcting device.

A paper by Kuzilin and Sintsov reports successfully applying this technique to a synthetic aperture telescope mirror (Kuzilin and Sintsov). However, the published paper contains inconsistencies between the text and the figures, and what work was actually performed and what results were actually achieved is unclear. No other system using a combination of holography and synthetic apertures is reported in the literature. On the other hand, beginning in 1966, numerous investigators describe systems which successfully correct aberrations in single aperture optics (Upatnieks).

Approach

This research consisted of three stages. Stage I developed a theoretical model for a simple, three-element, synthetic aperture system to determine how the wavefronts from the elements combine in the image plane. The model described the impulse response of the system.

This stage also developed how a hologram will operate on the image plane of the system. Stage II described how to construct and test a working system based on the three-element model. The purpose of this step was to verify the theoretic model by comparing predictions with observations. Finally, Stage III interpreted the empirical results in light of the developed theory.

II. Theory

Introduction

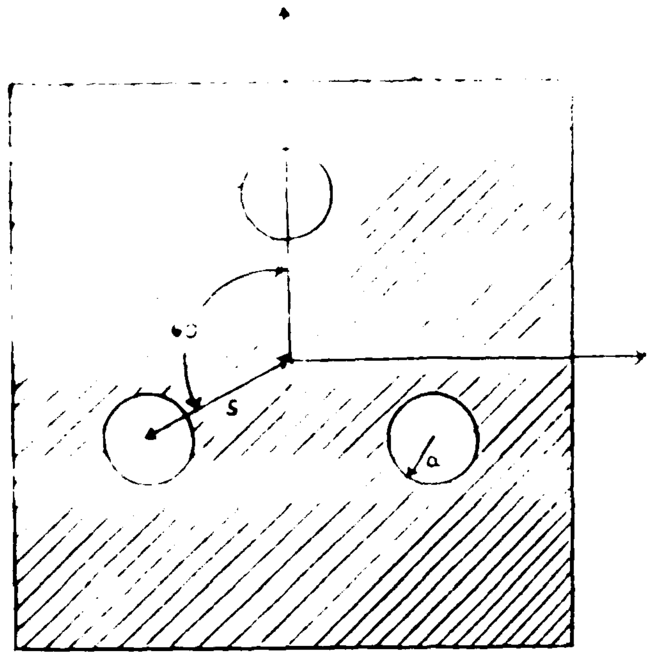
Developing the theory of how to optically correct aberrated wavefronts is straightforward. The chief difficulties are the algebraic manipulations are burdensome, and the developed terms are frequently cumbersome. However, by designing the system to take advantage of the mathematics, the burdens are reduced, and by rearranging the terms, the cumbersome expressions can be made elegant. The following discussion describes a simple system to offer a proof-of-concept of how holograms can correct aberrated wavefronts.

Systemic Arrangement

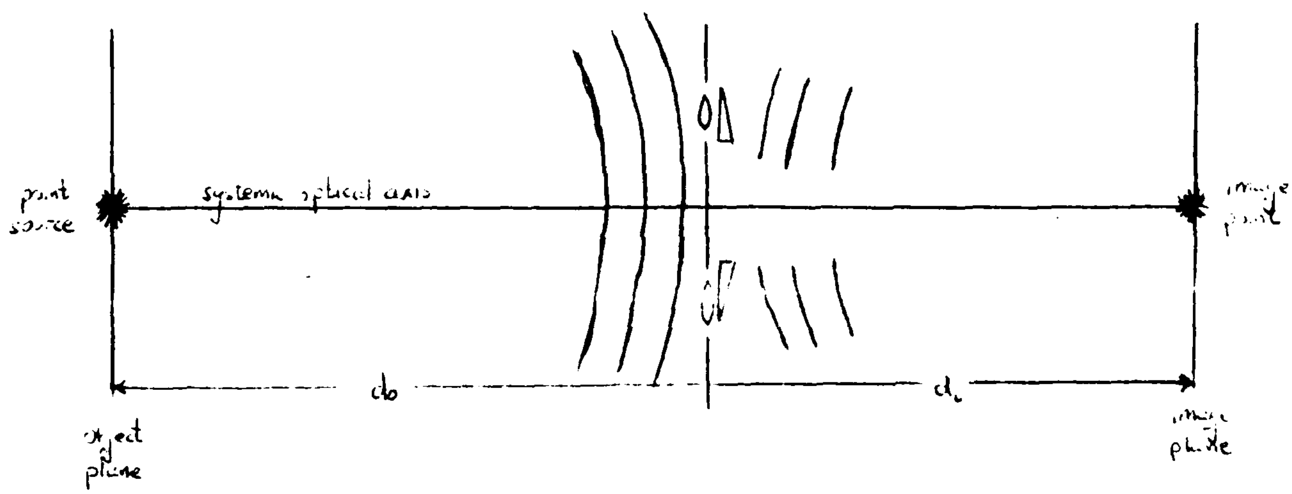
The synthetic aperture system considered here consisted of three converging lenses, each lens with focal length f and radius a, centered on the vertices of an equilateral triangle. The center of each lens was a distance s from the geometric center of the triangle, giving the triangle sides of length $s\sqrt{3}$. The optical axes of the lenses were parallel to each other, and the optical axis of the system is parallel to the optical axes of the lenses and intercepts the center of the equilateral triangle. Mirrors reflect the optical axes to form a common focal point. Conceptually, prisms immediately following the lenses may replace the mirrors. Figure 1 is a sketch of a front view and a side view of a one-dimensional analog of the system.

Point Spread Function of a Synthetic Aperture

The point spread function of a system is the image of a point source through the system and corresponds to the observed intensity of



Front View of the Synthetic Aperture



Side View of a One-Dimensional Analog of the Synthetic Aperture

Figure 1. A Synthetic Aperture System

the impulse response; the impulse response is the complex field amplitude in the image plane when a point object is in the object plane. To evaluate the point spread function, the system impulse response h and pupil function p(x,y) must be known.

Appendix A shows a derivation of the impulse response of a synthetic aperture. For a point source of wavelength λ at coordinates (x_0, y_0) in an object plane located d_0 to the left and parallel to the plane of the optical system, and an image plane with coordinates (x_1, y_1) located d_1 to the right and parallel to the plane of the optical system, the impulse response is

$$h = \frac{P(\xi, \eta)}{\lambda^2 d_0 f} \tag{1}$$

where $P(\xi,\eta)$ is the Fourier transform of the pupil function of the system, and ξ and η are spatial frequency coordinates defined by

$$\xi = \frac{d_0 x_1 + d_1 x_0}{\lambda d_0 d_1} \qquad \eta = \frac{d_0 y_1 + d_1 y_0}{\lambda d_0 d_1} \tag{2}$$

The pupil function accounts for the apertural effects of the finite size of individual elements in the synthetic aperture (see App A). Equation (1) describes the impulse response of a synthetic aperture only when the system is designed such that the apparent positions of the apertures as seen from the systemic focal point overlap the true positions (see App A).

The pupil function for the described system is

$$p(x,y) = \operatorname{circ}\left[\frac{x^2 + (y-s)^2}{a^2}\right]^{1/2} + \operatorname{circ}\left[\frac{(x+s\sqrt{3}/2)^2 + (y+s/2)^2}{a^2}\right]^{1/2} + \operatorname{circ}\left[\frac{(x-s\sqrt{3}/2)^2 + (y+s/2)^2}{a^2}\right]^{1/2} \tag{3}$$

Invoking the shift theorem, the Fourier transform of this pupil function is therefore

$$P(\xi,\eta) = \{\exp[j\pi\tfrac{1}{2}(\xi s\sqrt{3}-\eta s)]\cos[\pi\tfrac{1}{2}(\xi s\sqrt{3}+\eta s3)]$$

$$+ \exp[-j\pi\tfrac{1}{2}(\xi s\sqrt{3}+\eta s)]\cos[\pi\tfrac{1}{2}(\xi s\sqrt{3}-\eta s3)] \tag{4}$$

$$+ \exp[j\pi s\eta]\cos[\pi\xi s\sqrt{3}]\}\ \frac{a\ J_1(2\pi a\rho)}{\rho}$$

where $\rho^2 = \xi^2 + \eta^2$. Substituting (4) into (1) yields the impulse response for this system. The point spread function is then the modulus squared of the impulse response; therefore, $psf = h^*h$, or

$$psf = \{\cos^2[\pi s\tfrac{1}{2}(\sqrt{3}\xi+3\eta)] + \cos^2[\pi\tfrac{1}{2}(\sqrt{3}\xi-3\eta)] + \cos^2[\pi\xi\eta\sqrt{3}]$$

$$+ 6\cos[\pi s\tfrac{1}{2}(\sqrt{3}\xi+3\eta)]\cos[\pi s\tfrac{1}{2}(\sqrt{3}\xi-3\eta)]\cos[\pi\xi s\sqrt{3}]\} \tag{5}$$

$$\bullet\ \frac{\pi^2 a^4}{\lambda^2 d_0 d_1}\ \mathrm{somb}^2(2a\rho)$$

where $\mathrm{somb}(x) \equiv 2J_1(\pi x)/\pi x$.

Correcting Aberrated Wavefronts

The aberration free wavefront is spherical (Goodman, p 81). When a wavefront deviates from a spherical shape, it is termed *aberrated*. The aberrations arising from third-order deviations from a sphere are called Seidel aberrations (after Ludwig von Seidel) and may be approximated by a linear term. Thus an aberrated subject plane wavefront traveling in the yz-plane may be analytically described as

$$S(x,y) = s\ \exp[jk\phi(x,y)]\ \exp[jk(y - z)] \tag{6}$$

where s is the amplitude of the wave, ϕ describes the aberrations, and k is the wave number.

Suppose now that a reference plane wavefront of the form

$$R(x,y) = r\ \exp[jk(y + z)] \tag{7}$$

is introduced to the original subject wavefront. The waves will interfere with each other. If the intensity pattern $I = |S + R|^2$ of the interfering wavefronts is recorded, it will be of the form

$$I = r\ s\ \exp[-jK\phi(x,y)]\ \exp[j2kz] + 3 \text{ other terms} \tag{8}$$

Finally, suppose the original subject wavefront is allowed to fall on the recording of the interference pattern. The emerging wavefront will be of the form

$$SI = r\ s^2\ \exp\ [jK(y + z)] + 3 \text{ other terms} \tag{9}$$

This technique allows recovering one term which is an aberration free plane wavefront travelling in the same direction as the original reference wavefront.

If the interference pattern is recorded on a photographic plate, the actual intensity pattern will be [1 - I], where I is defined by (8). When the unaberrated wavefront is recovered, its field will be oppositely signed from the term in (9), and (9) will also contain four other terms. The additional terms are irrelevant except that they represent wasted energy, and the sign of the field is irrelevant when the intensity of the field is observed. If the wasted energy is a problem, photographic plates can be processed to modify the phase of the incident wavefronts, rather than the intensity, allowing more energy into the recovered term.

Summary

By using the theory developed in this chapter, both a system and a technique for using holograms to optically correct aberrated wavefronts can be synthesized. The next chapter discusses how a working system is constructed to allow exploiting the technique.

III. Experimental Apparatus

Introduction

The apparatus used to demonstrate the advantages of multiaperture systems was composed entirely of commercially available components. These components were selected based on availability, flexibility, and cost.

Arrangement

Figure 2 shows an overall schematic of the test system. The system was composed of four major subassemblies.

The laser assembly was a Spectra-Physics 2020 Argon ion laser (1) with multiline radiation from 450 nm to 515 nm. Output from the laser was coupled to a periscope (2). The periscope leveled the beam and aligned it to the optical axis of the system. One disadvantage of this laser was that its multiline radiation resulted in a too short coherence length (no intracavity etalon was available). Using a Michelson interferometer, the coherence length was empirically determined to be on the order of one centimeter. The implication of a short coherence length is that the path length differences between the subject and reference beams at the hologram must be measured precisely. To avoid this problem, the beam was passed through a grating to select the 514.5 nm green line. The coherence length of this line was measured and determined to be on the order of 30 cm.

The beam expander assembly was a two-stage device. The first stage was a standard spatial filter (3), using a microscope objective to focus the beam through a pinhole. Following the pinhole was a col-

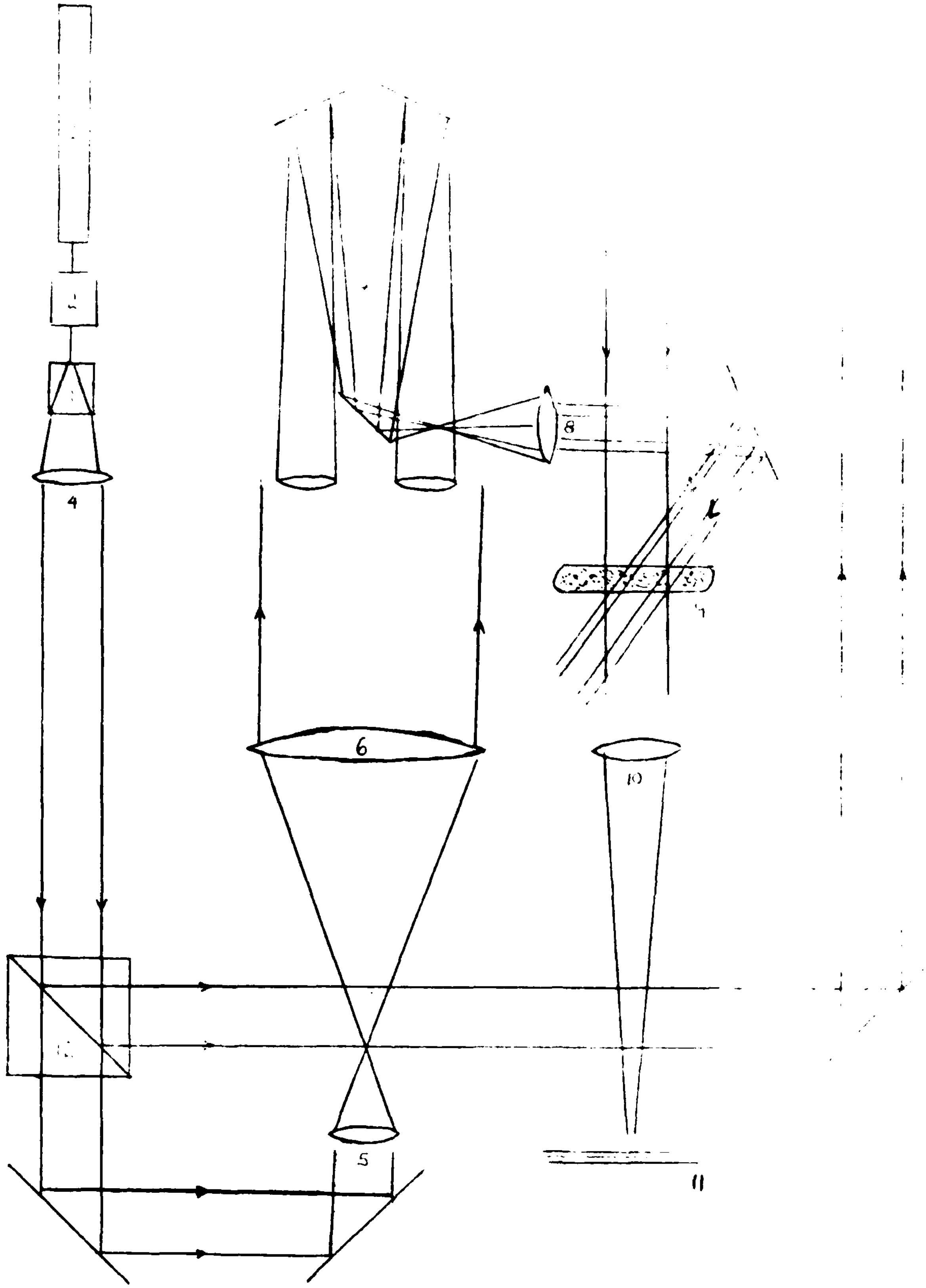


Figure 2. Schematic of the Experimental Arrang...

limating lens (4). These two elements expand the beam from approximately 4 mm to 25 mm. The second stage was a converging lens (5) followed be a second collimating lens (6) to enlarge the beam to approximately 240 mm.

The synthetic aperture assembly (7) consisted of three identical converging lenses (L1-L3), each lens with focal length f and radius a, centered on the vertices of an equilateral triangle. The center of each lens was a distance s from the geometric center of the triangle, giving the triangle sides of length $s\sqrt{3}$. The optical axes of the lenses were parallel to each other, and the optical axis of the system was parallel to the optical axes of the lenses and intercepted the center of the equilateral triangle. Mirrors (M1-M3) reflect the optical axes to form a common focal point. Only two of the three lenses and mirrors are shown in Fig 2 for clarity.

The holographic assembly consisted of a lens (8) to image the entrance pupil of the synthetic aperture onto the holographic plate (9). A second lens (10) focused the plate's image onto a screen (11). A beam splitter (12) sampled the beam illuminating the synthetic aperture. Mirrors directed the sampled beam onto the holographic plate.

Alignment

The components in this apparatus were carefully aligned to minimize aberrations. The technique used was to define a systemic optical axis and then to position the optical axis of each element, as defined by the Boys' points (Lipson and Lipson, p 455) of that element, to coincide with the systemic axis. Further details of this technique are in Taylor and Thompson.

IV. Results

Introduction

Results obtained from three synthetic aperture systems suggest the utility of the developed theory. The first system was a single aperture containing only a single lens. The second system was the triple aperture described in Chapter II; however the aperture was applied to a single lens. The final system was the full system described in Chapter II. The results here presented are qualitative, demonstrating utility, but requiring further quantitative investigation.

Single Aperture

The first demonstration was a single aperture containing only a single lens. The arrangement was as described in Fig 2 with Element 7 a single lens on axis with the system. The wavefront sampled for both the subject and reference legs was proven to be planar through both shear-plate interferometric analysis (Malacara, pp 105-48) and by observing the quality of the Airy pattern through a circular aperture.

Figure 3a shows the impulse response of the aperture and lens arrangement when the lens was aligned into the system. Notice the symmetry of the pattern and the similarity to an ideal Airy pattern. Figure 3b shows the impulse response of the aperture and lens arrangement when the lens was tilted and shifted out of alignment. The achieve the imperfect alignment, the lens was tilted so its optical axis met the system's axis at approximately 5-10 degrees, and then the lens was shifted to position the image approximately on the systemic optical axis. Figure 3c shows the impulse response of the holographically cor-

rected system. Observe that while the corrected response is not identical to the perfect response, it is unequivocally an improvement over the aberrated response. Recall also that the misalignments in the system are deliberately severe (to demonstrate utility); a practical system will have smaller errors to correct.

Triple Aperture

The arrangement and procedure are similar to the single aperture case. The only change is that the lens is now preceded by three small apertures at the vertices of an equilateral triangle. This arrangement simulated three perfectly aligned apertures.

Figure 4a shows the impulse response of the aperture and lens arrangement when the lens was perfectly aligned into the system. Notice the symmetry of the pattern. Figure 4b shows the impulse response of the aperture and lens arrangement when the lens was tilted and shifted out of alignment. Figure 4c shows the impulse response of the holographically corrected system. Again, observe that while the corrected response is not identical to the perfect response, it is unequivocally an improvement over the aberrated response.

Synthetic Aperture

The final aperture was the three-element synthetic aperture. This system was unique because of problems associated with aligning off-axis lenses and mirrors for minimum aberrations and adjusting the piston of the lenses to allow the focal points of the three legs to coincide. Eventually, a certain latitude of adjustment was accepted for demonstrating the system. Essentially, the system was aligned using an iterative combination of several alignment techniques.

Figure 5a shows the impulse response of the synthetic aperture arrangement when the aperture was reasonably well aligned into the system. Unlike the previous cases, no perfectly aligned system was available to show the impulse response of a perfect system. Notice the symmetry of the pattern and the clarity of the cosine modulation despite the cavalier approach to alignment. Figure 5b shows the holographically corrected impulse response of the synthetic aperture arrangement. Notice that the improvement is unsatisfying, but these results are only preliminary. More carefully aligning the elements into the system will certainly reduce the magnitude of aberrations the technique will have to correct, and then the improvement will be more dramatic. A quantitative assessment of the improvement would also allow assigning a number to describe the improvement.

Summary

The three demonstrations described qualitatively suggest that using holograms to optically correct aberrated wavefronts from synthetic apertures has merit. However, the demonstrations are only qualitative. Further investigations are needed to discover the quantitative improvements the technique may allow.

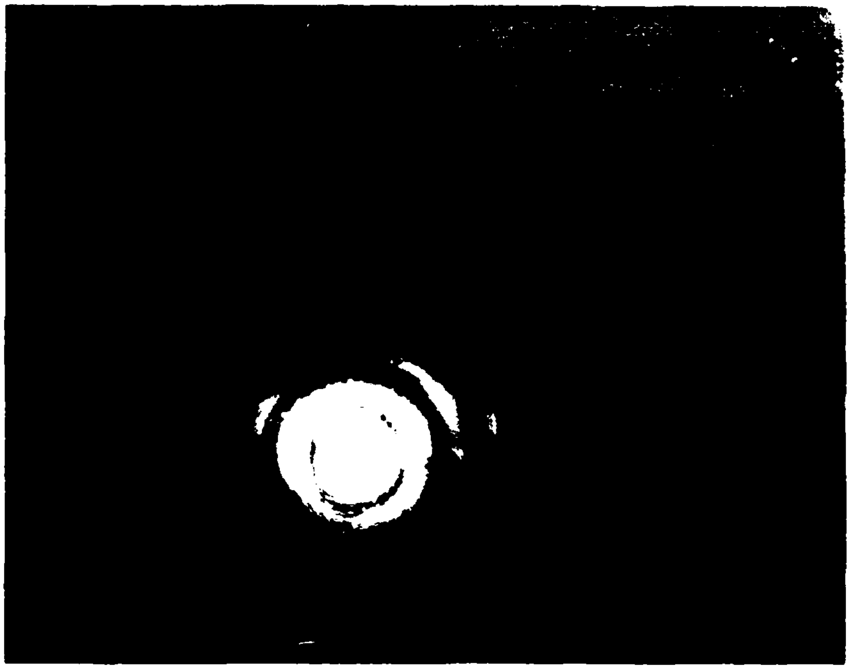


Figure 3a. Single Aperture Impulse Response--Unaberrated

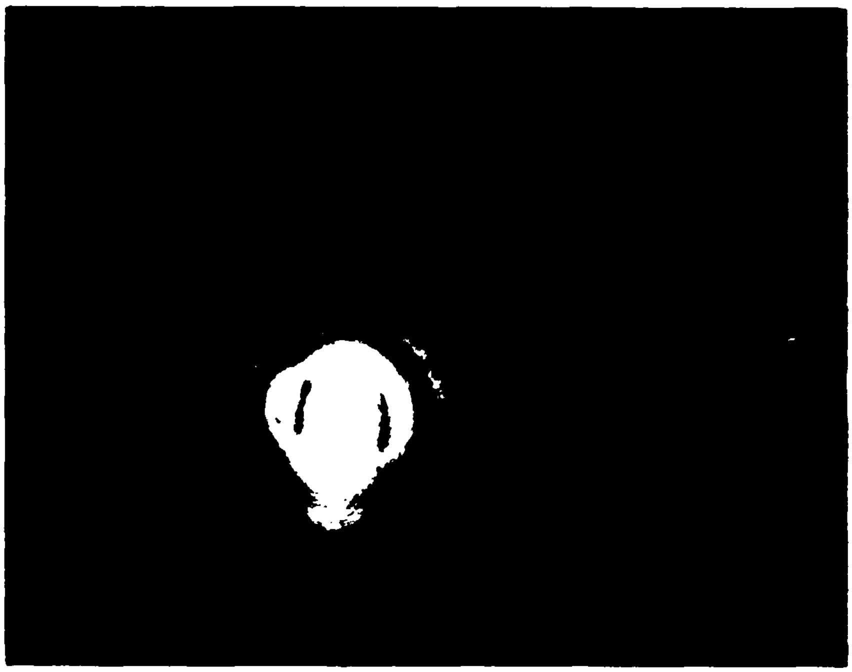


Figure 3b. Single Aperture Impulse Response--Aberrated

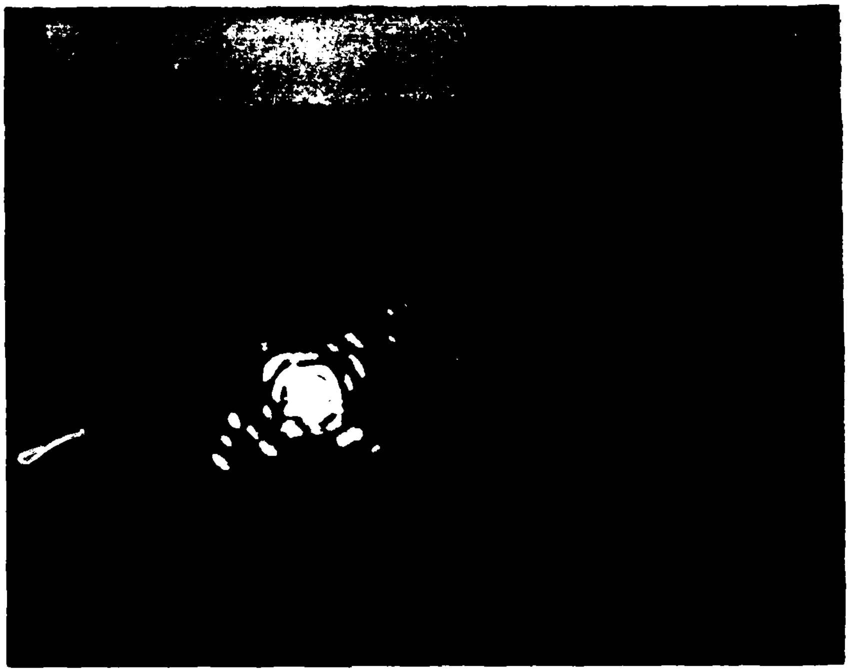


Figure 3c. Single Aperture Impulse Response--Corrected

Figure 4a. Triple Aperture Impulse Response--Unaberrated

Figure 4b. Triple Aperture Impulse Response--Aberrated

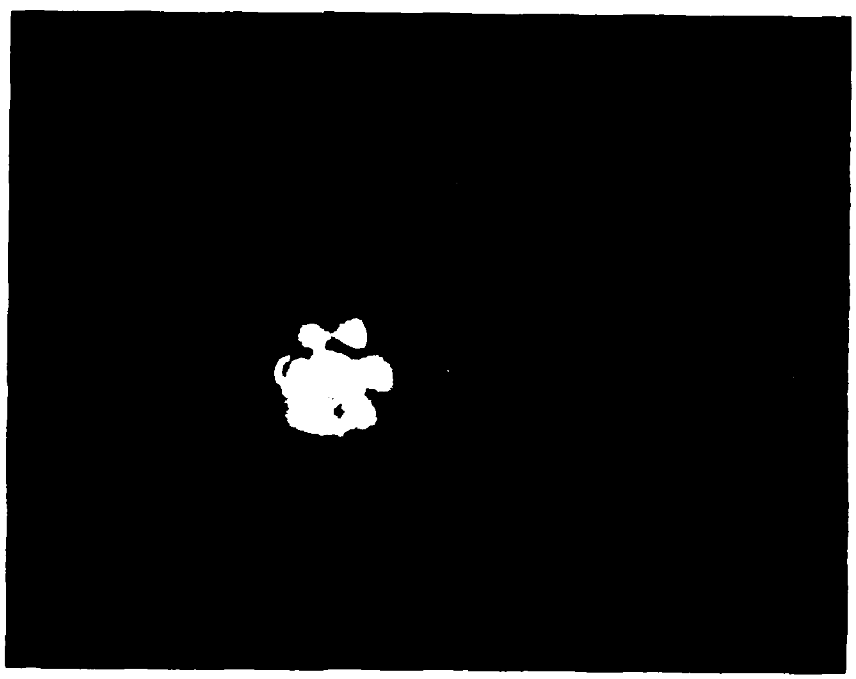


Figure 4c. Triple Aperture Impulse Response--Corrected

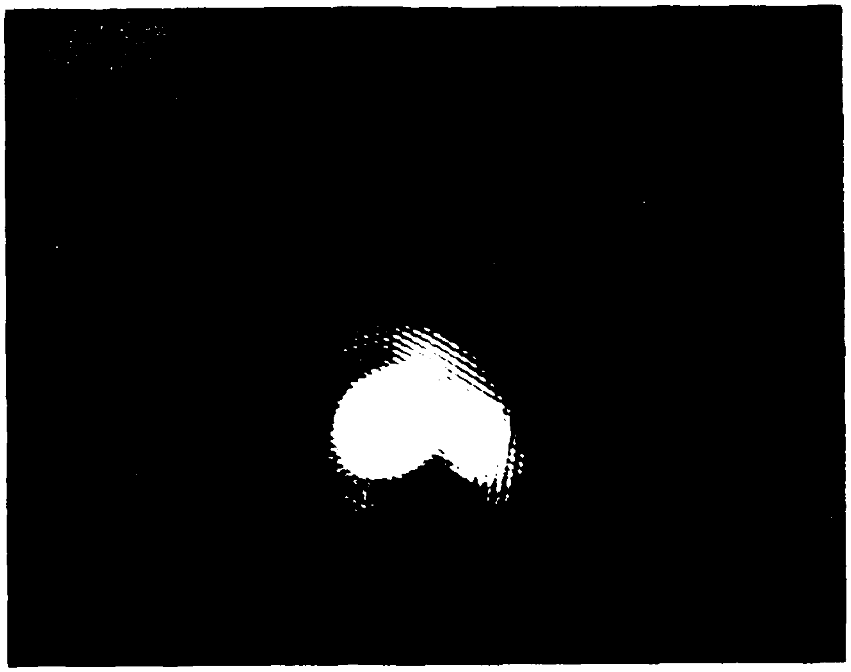


Figure 5a. Synthetic Aperture Impulse Response--Uncorrected

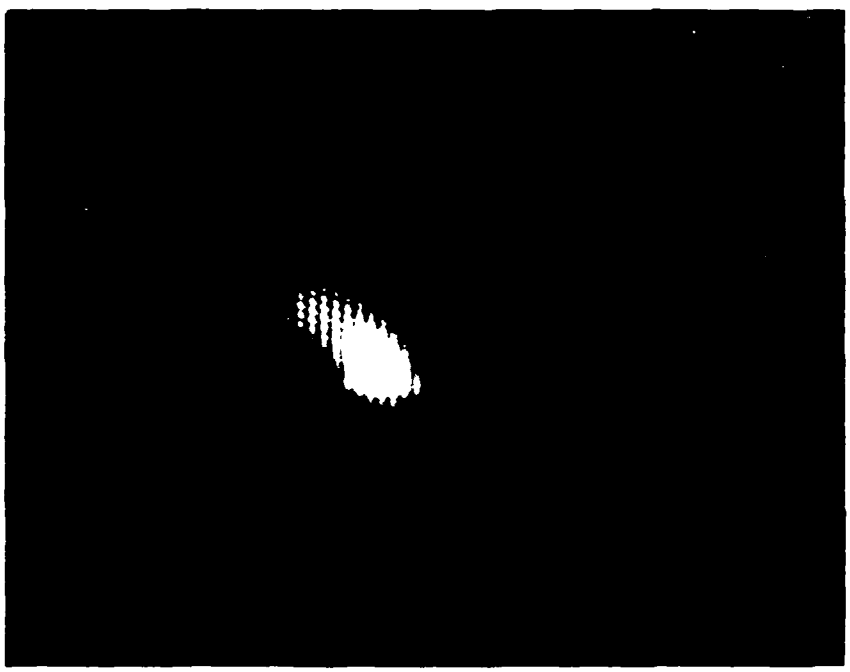


Figure 5b. Synthetic Aperture Impulse Response--Corrected

VI. Conclusions

Milestones

This thesis has produced two major products. First, the ideal point spread function of a specific synthetic aperture was derived. Second, the technique for using holograms to optically correct aberrated wavefronts from synthetic apertures has been qualitatively demonstrated for point sources.

Suggestions for Further Study

In the course of this research, several questions have appeared which remain unanswered. The questions divide into five areas.

The first area is that the theory of single aperture optics does not adequately address some issues in synthetic aperture optics. One unanswered issue is how to define a quantitative figure of merit for determining how accurately an image reproduces a given object. Typically texts require the image to approach the object except for a possible change in magnification; however, this qualitative approach does not allow evaluating similar systems on the basis of object fidelity. A quantitative figure of merit would allow objective evaluations of imaging systems. Another problem is defining a two-point resolution criteria for multiaperture systems. The cosine modulation of the point spread function renders the Rayleigh and Sparrow criteria ambiguous. A new resolution criteria must be defined to determine the resolving capabilities of synthetic aperture systems.

The second area is that some single aperture optical theory is already developed, but was not applied in this thesis. First, the

Gaussian theory used to describe the optics will not apply to optics more than 5-10 degrees from the systemic optical axis. While telescopic synthetic apertures--with elements close to the systemic axis--are readily described by Gaussian optics, other systems may require exact analysis to produce a model. Second, the derivation of the point spread function assumed elements positioned without error. Obviously such an assumption is ridiculous. The magnitude of error in the image plane due to mispositioned elements should be explored analytically (Hooker).

The third area is that some new systems are suggested by this research. The dimensions of the point spread function are related to the separation of the elements from each other. The optical axes of the elements are combined with a plane mirror, but a pyramidal mirror would allow the elements to be more closely positioned. Second, a faster method of recording the hologram would facilitate practical synthetic aperture systems. The current photographic plates are too slow with a potential repetition rate of about eight hours. Some nonlinear optical crystals offer a potential repetition rate of minutes or seconds. A high repetition rate is needed to accommodate potential systems which may vibrate or change their configuration in response to mechanical stresses (e.g. space-based telescopes). Third, the energy flow through the system needs analysis. The experimental system needs analysis. The experimental system suffers heavy energy losses, and a final design should consider the radiometric performance criteria. Finally the knowledge obtained from this research must be engineered into practical systems.

The fourth area is that a new technique must be devised for aligning off-axis elements. The current technique is to physically measure where an element should go with respect to the system's optical axis, and then place the element there. Unfortunately, the technique does not consider when the systemic axis is blocked by elements of the synthetic aperture, and in any event, the technique is sloppy compared to the techniques for aligning on-axis elements into the system. A technique must be devised to optically place the elements into the system.

Finally, a new technique of magnifying the point spread function must by devised. Currently microscope objective are used to enlarge the point spread function so the detail becomes obvious. Unfortunately the elements in the objectives allow too much light to suffer internal reflections, resulting in images with obvious interference patterns due to the multiple paths in the objectives. The interference patterns due bothersome, and eventually will interference with quantitative analysis of similar point spread functions. An objective with anti-reflective elements will have to be designed and built before quantitative measurements can be made.

Appendix A: Impulse Response of a Synthetic Aperture

Since phenomenon of propagating waves are linear, the image field $U_1(x_1,y_1)$ is related to the object field $U_O(x_O,y_O)$ through the convolution

$$U_1(x_1,y_1) = h(x_O,y_O;x_1,y_1) * U_O(x_O,y_O) \qquad \text{(A-1)}$$

where $h(x_O,y_O;x_1,y_1)$ is the impulse response at (x_1,y_1) due to a point source at (x_O,y_O). By describing the impulse response h of an optical system, the imaging properties of the system are determined.

To find h for a synthetic aperture, consider a point source at coordinates (x_O,y_O) in a plane located d_O to the left and parallel to the plane of the optical system. Spherical waves diverging from (x_O,y_O) will radiate from the point source. The paraxial approximation of those wave in the plane of the optical system is

$$U(x,y) = \frac{1}{j\lambda d_O} \exp\left[\frac{jk}{2d_O}\left[(x-x_O)^2 + (y-y_O)^2\right]\right] \qquad \text{(A-2)}$$

Determining how the optical system operates on incident waves requires first determining the complex pupil function $\tilde{p}(x,y)$ of the system.

The complex pupil function $\tilde{p}(x,y)$ is the complex transmittance of the optical system (Goodman, p 121). Generally, $\tilde{p}(x,y) = p(x,y)\cdot\exp[jkW(x,y)]$, where the factor $p(x,y)$, called the pupil function, accounts for apertural effects of the finite size of individual elements and the factor $\exp[jkW(x,y)]$, describes how the lenses modify the phase of the incident waves. For synthetic apertures, $W(x,y)$ has a term describing the effects of lenses and a second term describing the

effects of reflecting or refracting the optical axes of the elements to a common focal point. The term describing the phase transformation in the lens centered at (x_n, y_n) is simply (Goodman, p 81)

$$W_{lens}(x, y) = -\frac{(x-x_n)^2 + (y-y_n)^2}{2f} \qquad \text{(A-3)}$$

where f is the focal length of the lens. To determine the term describing the phase transformation due to mirrors or prisms operating on the optical axes of elements, consider that mirrors and prisms serve to translate and rotate the lenses from their actual position to an apparent position in a new coordinate system. Referring to Fig A-1, the lens in the new (primed) coordinate system is tilted with respect to the old (unprimed) coordinate system, leading to a term $W_{ref}(x,y) = -[(\tan\theta_x)x + (\tan\theta_y)y]$, where θ_x is the angle between the projection of the x' axis onto the xz-plane and the x axis, and θ_y is the angle between the projection of the y' axis onto the yz-plane and the y axis. Again referring to Fig A-1, $\tan\theta_x = x_n'/f$, and $\tan\theta_y = y_n'/f$, where (x_n', y_n') is the apparent position of the lens element, leading to a term

$$W_{ref}(x, y) = -\frac{x_n'x + y_n'y}{f} \qquad \text{(A-4)}$$

The error associated with setting $z_1 = f$ is obviated by designing a symmetric system so the error in placing z_1 is equal for all elements; the actual location of z_1 can be determined from Pythagorean's Theorem. Combining the apertural and lensing factors for a synthetic aperture composed of elements centered at (x_n, y_n) leads to a complex pupil function *for the special case when* $x_n' = x_n$ *and* $y_n' = y_n$:

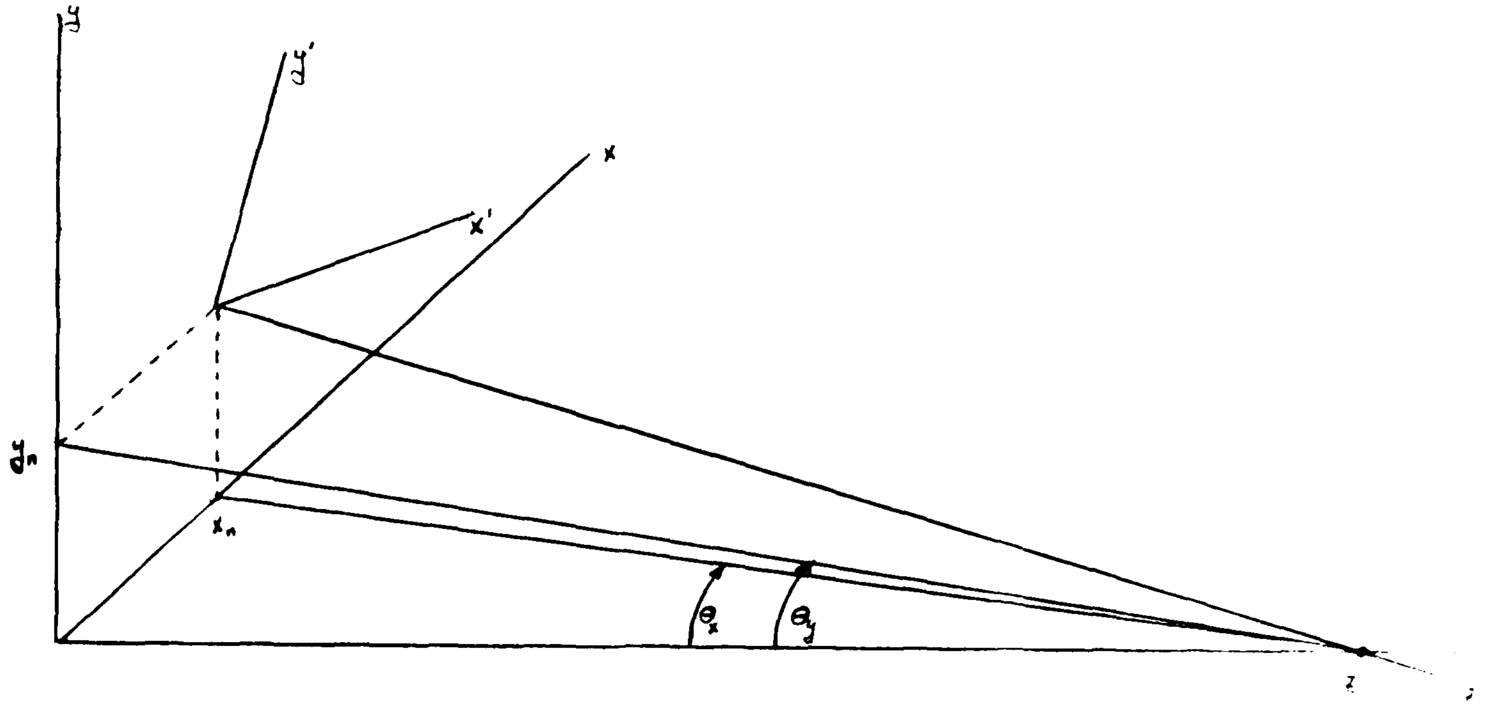


Figure A-1. The Relative Coordinate Systems

$$\tilde{p}(x,y) = \sum p(x-x_n, y-y_n) \exp\left[\frac{-jk}{2f}\left[x^2+x_n^2 + y^2+y_n^2\right]\right] \qquad (A-5)$$

This special case requires designing the system so the apparent positions of the lenses coincide with their actual positions. The wavefront U'(x,y) emerging from the elements is therefore the product of the spherical waves (A-2) and the complex pupil function (A-5)

$$U'(x,y) = U(x,y)\tilde{p}(x,y) \qquad (A-6)$$

The wave must now propogate to the image plane.

The Fresnel propagation formula allows following the wavefront to a plane located d_1 to the right and parallel to the plane of the optical system. The formula for finding $U_1(x_1,y_1)$ given U'(x,y) is

$$U_1(x_1,y_1) = \frac{1}{j\lambda d_1}\iint U'(x,y) \exp\left[\frac{jk}{2d_1}\left[(x_1-x)^2 + (y_1-y)^2\right]\right] dxdy \qquad (A-7)$$

or substituting for U'(x,y) and reducing

$$U_1(x_1,y_1) = \frac{1}{\lambda^2 d_o d_1}\exp\left[\frac{jk}{2d_o}(x_o^2+y_o^2)\right]\exp\left[\frac{jk}{2d_1}(x_1^2+y_1^2)\right]\exp\left[\frac{-jk}{}(x_n^2+y_n^2)\right]$$

$$\bullet \iint \sum p(x-x_n, y-y_n) \exp\left[\frac{jk}{2}\left[\frac{1}{d_o}+\frac{1}{d_1}-\frac{1}{f}\right]\left[x^2+y^2\right]\right] \qquad (A-8)$$

$$\bullet \exp\left[\frac{-j2\pi}{\lambda}\left[\left[\frac{x_o}{d_o}+\frac{x_1}{d_1}\right]x + \left[\frac{y_o}{d_o}+\frac{y_1}{d_1}\right]y\right]\right] dxdy$$

Choosing d_1 such that the Gaussian lens formula is satisfied, making the substitutions

$$\xi = \frac{d_o x_1 + d_1 x_o}{\lambda d_o d_1} \qquad \eta = \frac{d_o y_1 + d_1 y_o}{\lambda d_o d_1} \tag{A-9}$$

and recognizing $U_1(x_1,y_1)$ now defines $h(x_o,y_o;x_1,y_1)$, then

$$h = \frac{1}{\lambda^2 d_o d_1} \exp\left[\frac{jk}{2d_o}(x_o^2+y_o^2)\right] \exp\left[\frac{jk}{2d_1}(x_1^2+y_1^2)\right] \exp\left[\frac{-jk}{}(x_n^2 + y_n^2)\right] \tag{A-10}$$

$$\bullet \iint \sum p(x-x_n, y-y_n) \exp\left[-j2\pi\left[\xi x + \eta y\right]\right] dxdy$$

Equation (A-10) is the impulse response of a synthetic aperture system; however the leading exponential factors supply superfluous information.

Consider the three exponential factors in (A-10). For a well-behaved optical system, $x_o = x_1 d_o/d_1$ and $y_o = y_1 d_o/d_1$, which allows writing the first factor in terms of x_1 and y_1. When h as defined by (A-10) is used in convolution (A-1), all three exponential factors in h are independent of the convolution variables, therefore surviving the convolution operation unchanged. When the field U_1 from (A-1) is observed as $U_1^* U_1$, the three exponential factors vanish. Finally, defining $P(\xi,\eta)$ as the Fourier transform of $\Sigma p(x,y)$, the impulse response is

$$h = \frac{P(\xi, \eta)}{\lambda^2 d_o d_1} \tag{A-11}$$

where ξ and η are defined by (A-9). The impulse response (A-11) determines the imaging properties of the system. In particular, the impulse response my be used to find the point spread function of a specified system.

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Vita

Captain David A. Jacobs is a candidate for a Master of Science degree in engineering physics at the Air Force Institute of Technology (AFIT) at Wright-Patterson Air Force Base, Ohio. He entered AFIT in May 1985.

Captain Jacobs was born October 4, 1959 in Pittsburgh, Pennsylvania. He attended elementary and junior high schools in Harrisburg, Pennsylvania, and received a high school diploma from The Bolles School in Jacksonville, Florida. In 1981, he graduated from Worcester Polytechnic Institute with a Bachelor of Science degree in physics. Captain Jacobs' professional credentials include graduating from Squadron Officer School at Maxwell Air Force Base in 1984.

Receiving his commission through the Air Force Reserve Officers' Training Corps at the College of the Holy Cross in May 1981, Captain Jacobs was the Assistant Medical Physicist in the Department of Nuclear Medicine at the University of Massachusetts Medical Center while awaiting orders to active duty. He entered active duty in November 1981 as a physicist at the Air Force Weapons Laboratory (AFWL) at Kirtland Air Force Base, New Mexico.

Through May 1984, Captain Jacobs was the Laboratory's Radiological Physicist. He commanded teams recovering radioactive debris following flights of aerospace systems, and he executed safety analyses required for launch approval of aerospace systems carrying radioactive material. In May 1984, Captain Jacobs was selected to the Nuclear Weapon Concepts Group, a staff agency for the Director of Nuclear Technology. Here he directed studies of advanced weapons concepts to assess military utility and technological feasibility. While at AFWL, Captain Jacobs developed the analytic techniques used in the Air Force to evaluate how use of radioactive material on aerospace systems affects public health and the environment. He also developed the analytic tools used at AFWL to predict how thermonuclear weapons affect superhard targets. In March 1985, Captain Jacobs was appointed Acting Commander of the Kirtland Honor Guard, concurrently with his duties at AFWL.

Captain Jacobs is authorized the Senior Space Operations Badge. His decorations include the Air Force Achievement Medal with one oak leaf cluster. He is an expert marksman in both rifle and pistol. He is also an amateur/owner equestrian, and he and his appaloosa gelding are well known on the local show circuit.

Captain Jacobs' home is in Jacksonville, Florida.

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Abstract

This report [illegible] holographic optical element reduce aberrations in synthetic aperture optical systems. Using holograms to correct aberrated wavefronts from synthetic apertures provides a viable alternative to using active control systems to perfrom the same function. The holographic technique has applications to the Strategic Defense Initiative, surveillance and reconnaissance systems, and high technology manufacturing processes. A model for a three-element synthetic aperture system was developed to determine how the wavefronts from the elements combine in the image plane. The model includes developing the technique for introducing a holographic optical element to correct aberrations from individual elements in the aperture and from misalignments in the system. Based on this model, a proof-of-concept system was developed. Finally, the results of experiments to evaluate the point spread function of the empirical system are described. The report concludes that holographic optical elements can provide an alternative to control systems when combined with synthetic apertures to replace monolithic optics in applications requiring diffraction limited imaging systems.

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